21. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find (1) nine positive *integral numbers* in arithmetical progression the sum of whose squares is a *square number*; and (2) find nine integral *square numbers* whose sum is a *square number*.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

10. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Upon a surface one foot square, a coin one inch in diameter is thrown at random; what is the chance the coin *touches* or *intersects* both diagonals?

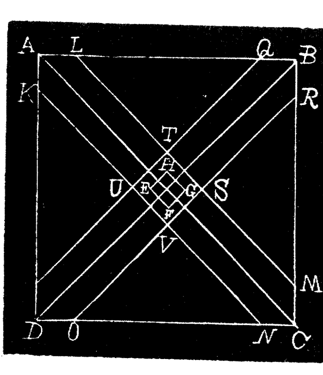
I. **Solution by Professor P. H. PHILBRICK, M. Sc., C. E., Lake Charles, Louisiana, and the PROPOSER.**

Represent the diagonals of the given square surface by AA' and BB'. If the center of the coin fall within the square $C_1C_2C_3C_4$, (C_1, C_2, C_3, and C_4 being the center of the coin when it is tangent to the diagonals), one of the following conditions is fulfiled: (1) the coin may *touch* both diagonals, (2) the coin may *intersect* both diagonals, (3) the coin may *touch* one diagonal and *intersect* the other. Since these conditions are *co-ordinate* with respect to the conditions of the problem, the total number of favorable chances is represented by the square $C_1C_2C_3C_4$. The total number of chances is represented by the area of the given square $ABA'B'$. Hence the required chance becomes $C=\frac{1}{144}$.

II. **Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Let $ABCD$ be the square. E, F, G, H, the centre of the coin when it is tangent to both diagonals.

Then if the centre of the coin falls any where on the square $EFGH$, it will either touch or fall upon both diagonals.



If the center falls any where on the area $AKUPDOVNCMSRBQTLA$, except when upon the square $FFGH$, it will touch or fall upon one diagonal. If the centre falls any where upon the areas KUP, OVN, MSR, QTL it will neither touch or fall upon a diagonal. Let A=area of $ABCD$=144 sq. inches.

a=area of $EFGH$=1 sq. inch.

b=area of $AKUPDOVNCMSRBQTLA$ —area $EFGH$. $\therefore$ $b=(48\sqrt{2}-9)$ sq. inches.

c=area $(KUP+OVN\times MSR+QTL)=(152-48\surd 2)$ sq. inches.

$\therefore\quad p$=chance that it touches or falls on both diagonals$=\dfrac{a}{A}=\dfrac{1}{144}$.

$\therefore\quad p_1$=chance that it touches or falls on one diagonal$=\dfrac{b}{A}=\dfrac{48\surd 2-9}{144}$.

$\therefore\quad p_2$=chance that it does not touch or fall on a diagonal$=\dfrac{c}{A}=\dfrac{152-48\surd 2}{144}$.

$\therefore\quad p+p_1+p_2=1$.

NOTE.—This problem was solved with different results by H. W. Draughon, Hon. Sosiah Drummond and ———. Professor Draughon and ———'s result is $\frac{1}{121}$. They suppose that the surface of the coin must be entirly on the given square, thus reducing the area of the surface upon which the centre of the coin may fall by a half-inch strip on each side.

Mr. D. consider the surface as though it were the bottom of a box. In this case, the area on which the coin could fall is $\left[144-\left(1-\frac{\pi}{4}\right)\right]$ square inches. Then the probability required is $\dfrac{4}{572-\pi}$. Each of these three results is right when viewed from the stand-point of its author, but we are doubtful whether the result $\frac{1}{121}$ can be harmonized with the theory of probability.

11. Proposed by ARTEMAS MARTIN, A. M., Ph. D., LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Find the average area of a triangle formed by joining a corner of a cube with any two points within the cube.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Take a lower corner O of the cube as the origin of co-ordinates, and let P_1 and P_2 be any two points taken at random within the cube. Make (x, y, z) and (u, v, w) the Cartesian triple co-ordinates of P_1 and P_2 respectively; then will $OP_1=\surd(x^2+y^2+z^2)$, and $OP_2=\surd(u^2+v^2+w^2)$.

Considering P_1 the *remoter* point with respect to the origin of co-ordinates, we have $P_1P_2=\surd[(x-u)^2+(y-v)^2+(z-w)^2]$; and consequently,

$$\cos\angle P_1OP_2=\cos\phi=\frac{(OP_1)^2-(OP_2)^2-(P_1P_2)^2}{2(OP_1)(OP_2)},=\frac{ux+vy+wz}{2(OP_1)(OP_2)}.$$

$$\therefore\quad \sin\phi=\surd(1-\cos^2\phi)=\frac{\surd[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2]}{2(OP_1)(OP_2)},\text{ and}$$

$$\triangle P_1OP_2=A=\tfrac{1}{2}(OP_1)(OP_2)\sin\phi=\tfrac{1}{4}\surd[(vx-uy)^2+(wx-uz)^2+(wy-vz)^2].$$

Hence the required average area becomes

$$\mathbf{A}=\frac{\int_0^a\int_0^a\int_0^a\int_0^x\int_0^y\int_0^z A\,dxdydzdudvdw}{\int_0^a\int_0^a\int_0^a\int_0^x\int_0^y\int_0^z dxdydzdudvdw},$$